Testing Revealed Preferences for Homotheticity with Two-Good Experiments: Supplementary Material

Jan Heufer *

*TU Dortmund University, Department of Economics and Social Science, 44221 Dortmund, Germany. E-mail: jan.heufer@tu-dortmund.de. Tel.: +49 231 755-7223. Fax: +49 231 755-3027.

1 An Illustration of the Homothetic Efficiency Index

The homothetic efficiency index in the paper is defined as

$$\mathrm{HE} = \min\left\{1, \min_{i,j\in\{1,\ldots,n\}} \{m_{ij}\}\right\},$$

where

$$m_{ij} = \frac{\mathbf{p}^i\mathbf{z}^j}{w^i}\frac{\mathbf{p}^j\mathbf{z}^i}{w^j}.$$

The observation-wise partial index is defined as

$$\tilde{\mathrm{HE}}(i) = \min\left\{1, \min_{j\in\{1,\ldots,n\}} \{m_{ij}\}\right\}.$$

The index is based on the fact that homotheticity implies that income expansion paths are rays through the origin. That implies that if $\mathbf{z}^2$ is demanded when facing the budget $B(\mathbf{p}^2, w^2)$, then the demand on the budget $B(\mathbf{p}^2, s\, w^2)$, with $s \in \mathbb{R}_+$, is $s\,\mathbf{z}^2$. Then if $\mathbf{z}^1 \in B(\mathbf{p}^2, s\, w^2)$, $\mathbf{z}^1$ must not be preferred to $s\,\mathbf{z}^2$.

Figure 1 illustrates the idea. Given that $\mathbf{z}^2$ is demanded on $B(\mathbf{p}^2, w^2)$, $([\mathbf{p}^2\mathbf{z}^1]/w^2)\mathbf{z}^2$ is demanded on $B(\mathbf{p}^2, ([\mathbf{p}^2\mathbf{z}^1]/w^2)w^2) = B(\mathbf{p}^2, \mathbf{p}^2\mathbf{z}^1)$. Thus, if the consumer had homothetic preferences, $([\mathbf{p}^2\mathbf{z}^1]/w^2)\mathbf{z}^2$ would give at least the same utility as $\mathbf{z}^1$. But at prices $\mathbf{p}^1$, where $\mathbf{z}^1$ was demanded at expenditure w^1, $([\mathbf{p}^2\mathbf{z}^1]/w^2)\mathbf{z}^2$ was also affordable; in fact, the consumer could have purchased it at expenditure $\left[(\mathbf{p}^2\mathbf{z}^1)/w^2\right]\left[\mathbf{p}^1\mathbf{z}^2\right] < w^1$.

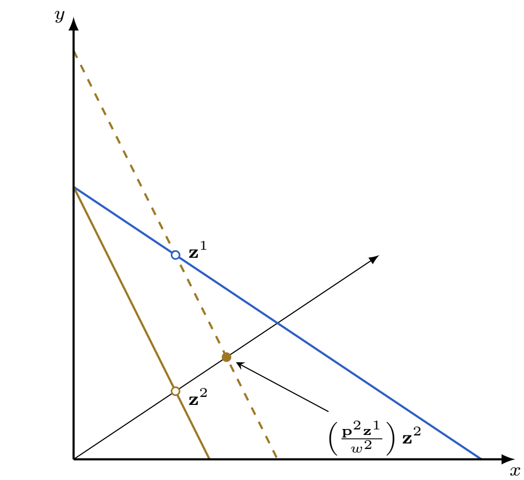
2 The Power of a Test

Bronars (1987) suggested a Monte Carlo approach to determine the power the test has against random behaviour. The approximate power of the test is the percentage of random choices which violated GARP. For the simple case of just two observation, Bronars showed analytically that the probability of a violation of GARP, given random choice, is highest when two intersecting budgets are nearly parallel such that the length (or area) of a budget lying inside the other budget is large relative to the rest of the budget. Knowing the corresponding conditions for HARP would make it easier for researchers to design powerful budget combinations. The following proposition will be helpful:

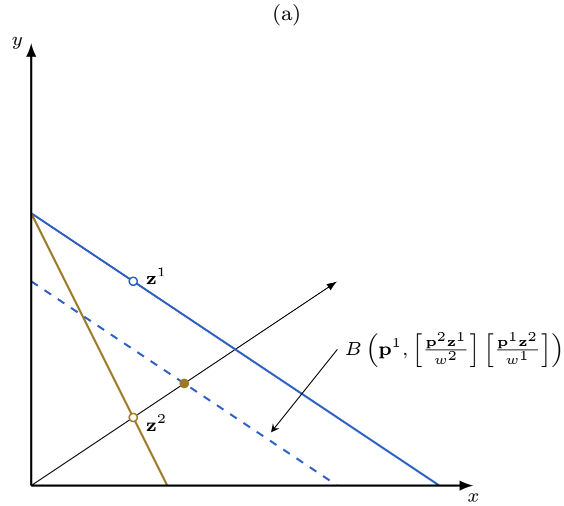
Proposition 1. *The following conditions are equivalent:*

(1) The set $\{(\mathbf{z}^i, \mathbf{p}^i, w^i)\}_{i=1}^n$ *satisfies HARP;*
(2) The set $\{(\mathbf{z}, \mathbf{p}, w) : \tau \geq 0$ *and* $[\mathbf{z} = \tau\,\mathbf{z}^i, \mathbf{p} = \mathbf{p}^{\mathbf{i}}, w = \tau\, w^i]$ *for some* $i \in \{0, 1, \ldots, n\}\}$ *satisfies GARP.*

Proof of Proposition 1. Generate a new observation $(\hat{\mathbf{z}}^i, \hat{\mathbf{p}}^i, \hat{w}) = (\tau\,\mathbf{z}^i, \mathbf{p}^i, \tau\, w^i)$ for some $\tau > 0$.



(a)



(b)

Figure 1: Illustration of the homothetic efficiency index.

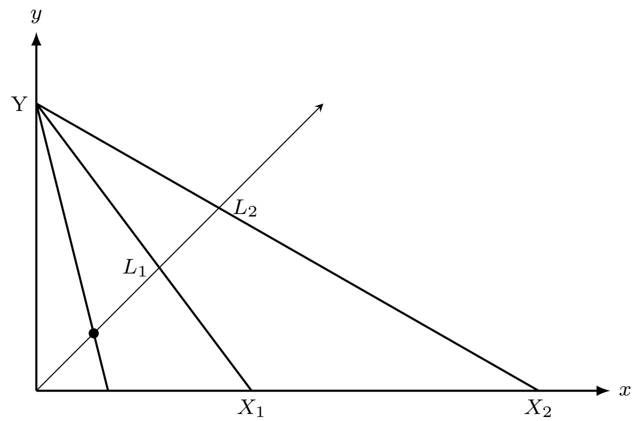


Figure 2: The probability of observing a violation of HARP.

If $\tau \geq t^{i,j}$, then $\tau\,\mathbf{z}^i H^0 \mathbf{z}^j$. Using Eq. (1) in the paper, this implies that $t^{i,j}\,\mathbf{z}^i$ and therefore $\tau\,\mathbf{z}^i$ is revealed preferred to $\mathbf{z}^j$ via a chain of bundles: $t^{i,k}\,\mathbf{z}^i R\,\mathbf{z}^k$, $t^{k,\ell}\,\mathbf{z}^k R\,\mathbf{z}^\ell$, ..., $t^{m,j}\,\mathbf{z}^m R\,\mathbf{z}^j$ for some sequence of indices $k, \ell, \ldots, m$. Thus $\left[\text{not } \mathbf{z}^j P^0 t^{i,j}\,\mathbf{z}^i\right]$ (HARP) is equivalent to $\left[\text{not } \mathbf{z}^j P^0 \hat{\mathbf{z}}^i\right]$ (GARP) for all $\hat{\mathbf{z}}^i = \tau\,\mathbf{z}^i$ with $\tau > t^{i,j}$.

If $\tau < t^{i,j}$, then $\left[\text{not } \tau\,\mathbf{z}^i H^0 \mathbf{z}^j\right]$ and $\left[\text{not } \hat{\mathbf{z}}^i R\,\mathbf{z}^j\right]$. Thus neither HARP nor GARP impose a condition on the relation between $\hat{\mathbf{z}}^i$ and $\mathbf{z}^j$. □

Proposition 1 implies that a set of budgets which is powerful for a GARP test will also be powerful for a test for homotheticity. Additionally, the budgets for a HARP test do not even have to intersect; it is sufficient for the budgets to have similar slopes. Figure 2 illustrates this: Conditional on the observation on the budget with the steepest slope, the probability of a violation of HARP, given random choice, will be higher when this budget is paired with another budget with a similar slope. This is because homotheticity implies that the choice on the other budgets is made to the lower right side of the indicated expansion line, and the ratio $||\overline{L_i\,Y}||/||\overline{L_i\,X_i}||$ decreases as the slope decreases.

3 More Details on the Data Analysis

3.1 Robustness Check for Parametric Estimations

Andreoni and Miller (2002) excluded three subjects from the parametric analysis in which they estimated a CES utility function. These subjects had AEIs below the 95% threshold. They found that 62 of the 142 subjects in the sessions with eight budgets strongly fitted a prototypical utility function. The remainder was used for the estimation, except for the three subjects with low efficiency (i.e., 77 subjects from the eight budgets sessions where used). As all subjects who strongly fit a prototypical utility function satisfy PHARP, this means that 48 out of 77 subjects (= 61.04%) which were used for the estimation violate PHARP. However, most of them have rather high levels of homothetic efficiency, as indicated in Figure 1.(a) in the main part of the paper.

3.2 Gender Differences

Table 1 shows the consistency with WGARP and PHARP in Andreoni and Vesterlund's (2001; also Andreoni and Miller 2002) data by gender of subjects. Note that the PHARP implies but is not implied by the WGARP.

It is interesting to note that, at the 5% significance level, male subjects are significantly less likely than female subjects to violate the PHARP ($\chi^2 = 5.175$, $p = 0.0229$), but not WGARP ($\chi^2 = 2.782$, $p = 0.0953$). Andreoni and Vesterlund's finding that male subjects are more price sensitive than women might explain this result to some extent. Note that homotheticity implies that the slope of rays through the origin and a demand bundle increases in the price ratio. Suppose two subjects try to maximise well behaved homothetic utility functions, but make minor errors in specifying their demand. Furthermore, suppose that the first subject mostly specifies demand near the 45° line (i.e., she is not price sensitive), while the second subject specifies demand near one of the two axes for extreme price ratios and around the 45° line for price ratios near 1. Then the slope of the rays are all similar for the first subject, but they can be quite different for the second subject. While in both cases minor errors can lead to a violation of HARP, this is more likely to happen to the first subject. Thus, some subjects may be somewhat more 'violation-prone' than others.

The fact that female subject violate WGARP more frequently than male subjects alone cannot explain the difference in the PHARP-violations, as a χ^2 test shows that the difference in WGARP-violations is not significant. Furthermore, if female subjects had homothetic preferences and would only make minor errors in specifying demand, this would show in the distribution of the homothetic efficiency index, which then should be close to one for female subjects. This is not the case: Figure 3 shows the distribution of the HE of those subjects who violate PHARP. The distribution is quite similar for both genders, with more male subjects having a HE greater than .95 than female subjects.

Two of the three subjects which were excluded in the parametric analysis because of their low efficiency level were female. Excluding the three subjects for the analysis of gender differences in homothetic efficiency changes the results slightly, but the difference remains significant ($\chi^2 = 4.334$, $p = 0.0374$).

		PHARP		WGARP		ALL
		SATISFIED	VIOLATED	SATISFIED	VIOLATED	
SEX	MALE	67	28	89	6	95
	FEMALE	24	23	40	7	47
ALL		91	51	129	13	142

Table 1: Subjects who satisfy PHARP and WGARP in the dictator game of Andreoni and Vesterlund (2001) and Andreoni and Miller (2002).

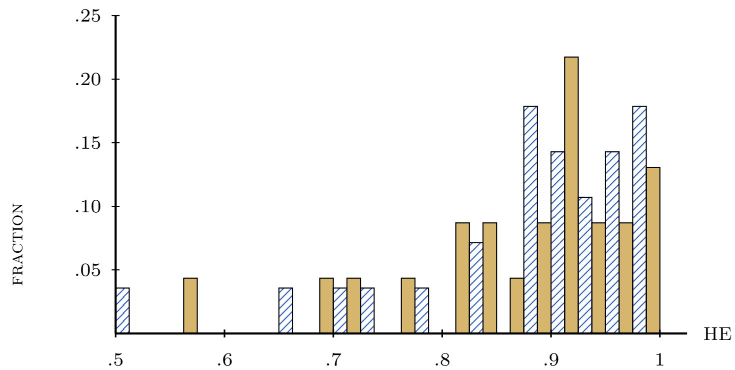
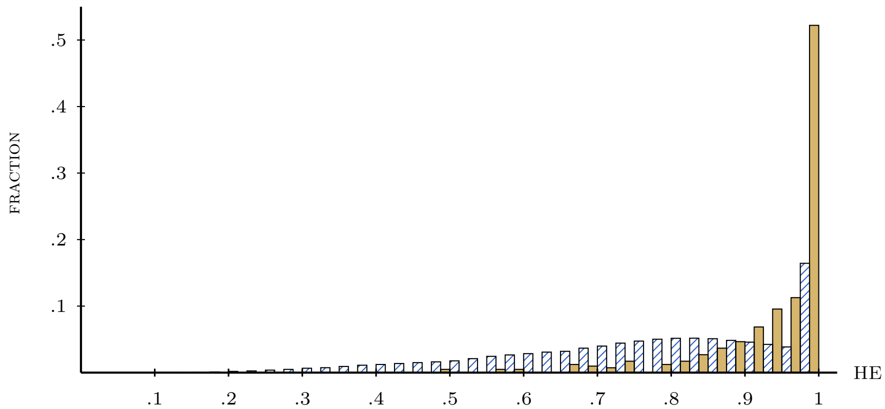


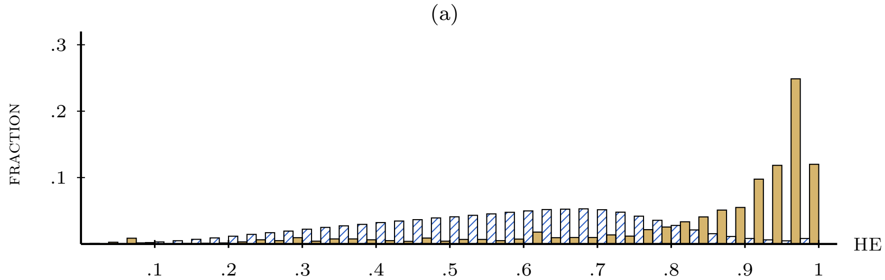
Figure 3: shows the distribution for male, for female subjects. Distribution of the homothetic efficiency index for subjects which violate PHARP.

3.3 Observation-Wise Homothetic Efficiency

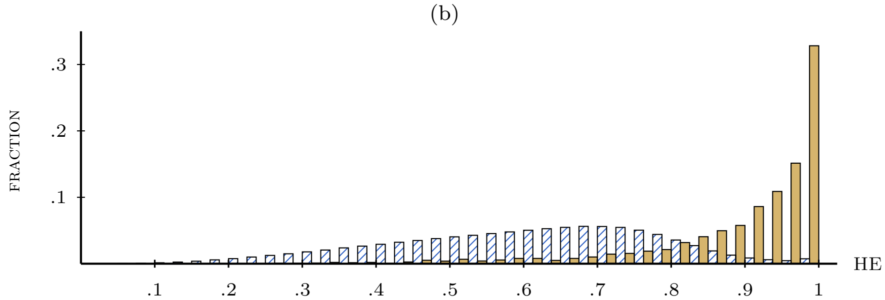
Figure 4 shows the distribution of the $\tilde{\mathrm{HE}}(i)$ in the experimental data and the random choice sets. It does show that the low HE values of both subjects and random choice sets are mainly due to a small fraction of individual choices. But it also reveals that in the FKM and CFGK data the majority of individual choices exhibit homothetic inefficiency. For the AM data, which only shows the $\tilde{\mathrm{HE}}(i)$ for those subjects which violate PHARP, many values are 1 or close to 1, but still many observations exhibit inefficiency. Also note that here many random choices have a HE level of 1.



(a)



(b)



(c)

Figure 4: ▨ shows the distribution for random choices, ■ for actual subjects. Distribution of the $\tilde{\mathrm{HE}}(i)$ and the Monte-Carlo results for the data of (a) Andreoni and Vesterlund (2001) and Andreoni and Miller (2002), (b) Fisman et al. (2007), (c) Choi et al. (2007a). The dashed lines indicates the HE which exceeds the HE of 95% of the data sets in the Monte Carlo simulation.

References

Andreoni, J. and Miller, J. (2002): "Giving According to GARP: An Experimental Test of the Consistency of Preferences for Altruism," *Econometrica* 70(2):737–753.

Andreoni, J. and Vesterlund, L. (2001): "Which is the Fair Sex? Gender Differences in Altruism," *Quarterly Journal of Economics* 116(1):293–312.

Bronars, S. G. (1987): "The Power of Nonparametric Tests of Preference Maximization," *Econometrica* 55(3):693–698.

Choi, S., Fisman, R., Gale, D., and Kariv, S. (2007a): "Consistency and Heterogeneity of Individual Behavior under Uncertainty," *American Economic Review* 97(5):1921–1938.

Fisman, R., Kariv, S., and Markovits, D. (2007): "Individual Preferences for Giving," *American Economic Review* 97(5):1858–1876.