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Stochastic Revealed Preference and Rationalizability

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Abstract This article explores rationalizability issues for finite sets of observations of stochastic choice in the framework introduced by Bandyopadhyay et al. (Journal of Economic Theory, 84, 95–110, 1999). It is argued that a useful approach is to consider indirect preferences on budgets instead of direct preferences on commodity bundles. A new rationalizability condition for stochastic choices, "rationalizable in terms of stochastic orderings on the normalized price space" (RSOP), is defined. RSOP is satisfied if and only if there exists a solution to a linear feasibility problem. The existence of a solution also implies rationalizability in terms of stochastic orderings on the commodity space. Furthermore it is shown that the problem of finding sufficiency conditions for binary choice probabilities to be rationalizable bears similarities to the problem considered here.

Keywords Stochastic choice · Rationalizability · Revealed preference · Weak axiom of stochastic revealed preference · Revealed favorability

1 Introduction

Bandyopadhyay, Dasgupta, and Pattanaik (1999) (henceforth BDP) initiated a line of investigation in which they explored choice behavior of a consumer who chooses in a stochastic fashion from different budget sets. In BDP (2002) this approach was extended by an interpretation of tuples of deterministic demand functions of different consumers as a stochastic demand function. They define a weak axiom of stochastic revealed preference which is implied by but does not imply rationalizability in terms of stochastic orderings on the commodity space.[1] In BDP (2004), the authors note that

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[1] Formal definitions are given in Section 2.

> it is not at all obvious what would be a natural stochastic translation of the familiar strong axiom of revealed preference and what would be the implications of such a 'strong axiom of stochastic revealed preference'.

It is the purpose of this article to explore rationalizability issues, provide a condition for rationalizability in terms of stochastic orderings, and to discuss related problems.

While stochastic preferences and choices are not a new topic, it continues to receive considerable attention. An early contribution was made by Falmagne (1978), who was the first to find conditions for rationalizability of stochastic choices by a probability distribution over linear preference orderings. Barberá and Pattanaik (1986) introduced Falmagne's result to the economics literature; they model preferences as orderings instead of "utility scales" and draw parallels to the revealed preference literature. Nandeibam (2009) examines rationalizability of a choice system in a general choice context, with no restrictions on the universal set of alternatives. Alcantud (2006) and Dasgupta (2009) extend the framework of single valued stochastic demand to possibly multi valued stochastic demand correspondences. Dasgupta (2005) introduces a consistency postulate for input-output choices by firms and generalized the approach to stochastic supply functions in Dasgupta (2009a). Contributions such as Falmagne (1978) and Nandeibam (2009) are rather abstract approaches, considering choice probabilities on choice sets which can be somewhat arbitrary collections of alternatives. BDP consider a framework more commonly used in economics, i.e. the budgets faced by a competitive consumer.

Two interpretations of the framework BDP and of stochastic choice in general need to be considered. The first one is about a single consumer who has random preferences and, as a preference maximizer, will therefore display random decisions. The second one is about a set of consumers each of whom has a deterministic preference, and whose decisions are made anonymously, in which case the observer can model the observed demands as if they came from a single consumer with random preferences. The first interpretation is in the focus of BDP (1999), the second interpretation is presented in BDP (2002).

For the first interpretation, suppose a consumer specifies a probability for each subset of a given budget such that the probability assignments add up to unity. Suppose further that we observe these probability assignments on a finite set of budgets. Can we find conditions on the probability assignments such that, if these conditions are satisfied, we cannot reject the hypothesis that the consumer has random preference orderings and, given the budget set, optimizes on the basis of his realized preference ordering?

For the second interpretation, suppose we observe single choices of many anonymous consumers on a finite set of budgets, such that we observe each individual decision but do not know by which consumer the decision was made. Can we find conditions on the choices such that, if these conditions are satisfied, we cannot reject the hypothesis that the choices were made by a set of maximizing consumers?

The problem is complicated by at least two factors. Firstly, in the context of stochastic revealed preference, budget sets are infinite sets of alternatives. The stochastic choice literature is usually confined to choices from finite sets. Falmagne (1978) explicitly confines himself to choices from finite sets of alternatives. Cohen (1980) extends Falmagne's approach to the case of an infinite overall set of alternatives, but again, all choice sets are finite subsets of the set of alternatives. A recent exception is Nandeibam (2009).

Secondly, even in the deterministic case we are not in general able to recover the entire ranking of a consumer with only a finite set of observations. This is simply because a consumer might choose a in a situation where b is not available, and chooses b in a situation where a is not available. If there are no further observations which can be used to deduce a relation between a and b via a chain of other choices, we do not know if the consumer prefers a over b. In the stochastic case we are therefore only able to deduce minimal choice probabilities; for example, we might be able to deduce that the consumer prefers a over b in at least 30% of all cases and b over a in at least 20% of all cases.

It will be argued that a useful way to understand and analyze stochastic choices on standard budget sets is in terms of indirect preferences on the price-income space or the normalized price space. To this end Sakai's (1977) conditions for indirect preferences from which a utility function can be deduced are used. That is, the problem of finding a probability measure on orderings over the available commodity bundles is transformed into the problem of finding a probability measure on orderings over the budgets on which choices are observed.

It is also shown that the rationalizability problem bears similarities to the problem of finding necessary and sufficient conditions for rationalizability of binary choice probabilities; this is specifically true for stochastic revealed preference conditions based on partial relations between alternatives. That is, a set of conditions sufficient for rationalizability is likely to also be applicable to the strand of literature concerned with binary choice. No finite sets of necessary and sufficient conditions for each number of alternatives is known, and Fishburn (1990) showed that the set of conditions on the choice probabilities that are sufficient for rationalizability regardless of the number of alternatives must be infinite. This poses some problems for the framework considered here.

The remainder of the article is organized as follows. Section 2 introduces the notation, and recalls the relevant work by BDP and Sakai. Section 3 introduces a linear feasibility problem which is solvable if and only if the choices are rationalizable in terms of stochastic orderings on the normalized price space. It also implies the existence of probability distribution of orderings on the commodity space. Problems, in particular with connection to binary choices, are discussed. Section 4 concludes.

2 Preliminaries

2.1 Notation and Basic Concepts

First the primitive elements of the framework for deterministic revealed preference are introduced. It is assumed that we observe the demand of a consumer facing a finite number of competitive (i.e. linear) budgets. That is, for each budget, we observe the commodity bundle or bundles demanded by the consumer given his income (or wealth) and the price vector.

Let ℓ be the number of commodities, and let $X = \mathbb{R}^{\ell}_{+}$ be the *commodity space*.[2] The *normalized price space* P is defined by

$$P = \big\{p : p = (p_1, p_2, \ldots, p_\ell) \text{ and } p_i = \rho_i / w \quad (i = 1, 2, \ldots, \ell) \\ \text{for some } (\rho_1, \rho_2, \ldots, \rho_\ell, w) \in \mathbb{R}^{\ell}_{++} \times \mathbb{R}_{++}\big\},$$

where ρ_i denotes the price of commodity i and w denotes the consumer's income. We assume that we observe consumption decisions on a finite set of n budgets. A *budget set* can then be defined by $\{x \in X : px \leq 1\}$. We will denote the budget sets as $B^i = B(p^i)$ and the upper

[2] Notation: $\mathbb{R}^{\ell}_{+} = \{x \in \mathbb{R}^{\ell} : x \geqq 0\}$, $\mathbb{R}^{\ell}_{++} = \{x \in \mathbb{R}^{\ell} : x > 0\}$, where " $x \geqq y$ " means " $x_i \geq y_i$ for all i ", and $x \neq y$ ", and " $x > y$ " means " $x_i > y_i$ for all i ". Note the convention to use subscripts to denote scalars or vector components and superscripts to index bundles.

bound of budget sets as $\bar{B}^i = \{x \in X : p^i x^i = 1\}$, where superscripts index the observation. Furthermore $\mathscr{B} \subset 2^X$ denotes the family of all budget sets, i.e. $\mathscr{B} = \{B(p) : p \in P\}$.

Let $h : \mathscr{B} \to X$ be a non-empty *demand function* on $\mathscr{B}$ which assigns to each B a non-empty subset $h(B)$, and let H be the set of all demand functions. Unless otherwise stated, we assume that each $h(B)$ is a singleton, and denote $x^i = (x^i_1, x^i_2, \ldots, x^i_\ell) = h(B^i)$. Furthermore we shall assume that the entire income is spent, that is, $h(B^i) = h(\bar{B}^i)$.

If $p^i x^i \geq p^i x$ we say that the observation x^i is *directly revealed preferred* to x. In this case, the pair $\{x^i, x\}$ is an element of the revealed preference relation $R \subseteq X \times X$. For brevity, we write $x^i R x$. The observation x^i is *revealed preferred* to x, written $x^i R^* x$, if either $x^i R x$ or for some sequence of bundles $(x^j, x^k, \ldots, x^m)$ it is the case that $x^i R x^j$, $x^j R x^k$, …, $x^m R x$. In this case R^* is the *transitive closure* of the relation R, i.e. the smallest transitive relation which contains R.

An *ordering* in the sense of BDP is a binary relation $\succsim$ over X satisfying: (i) reflexivity: for all $x \in X$, $x \succsim x$; (ii) connectedness (or completeness): for all distinct $x, y \in X$, $x \succsim y$ or $y \succsim x$; and (iii) transitivity: for all $x, y, z \in X$, $[x \succsim y$ and $y \succsim z]$ implies $x \succsim z$. The set of all orderings on X will be needed for the definition of rationalizability in terms of stochastic orderings. Let $\mathscr{R}$ denote the set of all orderings over X. We will denote the elements of $\mathscr{R}$ as $\succsim^R$ to distinguish them from orderings on the price space defined in Section 2.2.

The *weak axiom of revealed preference* (WARP) asserts that R is asymmetric: For all $x, x' \in X$, $x \neq x'$, $x R x'$ implies [not $x' R x$]. The *strong axiom of revealed preference* (SARP) asserts that the transitive closure of R, R^*, is asymmetric: $x R^* x'$ implies [not $x' R^* x$].

Note that the revealed preference relation R is empirically defined and thus, with only finitely many observations, not an ordering because it is not complete. However, Newman (1960) showed how a complete ordering can be derived from a demand function which satisfies SARP; thus we know that if SARP is satisfied, an ordering on the commodity space consistent with the observed demand exists.

2.2 Indirect Revealed Preference and Revealed Favorability

There is a notion of *indirect revealed preference* due to Sakai (1977), Little (1979), and Richter (1979).[3] We will rely on Sakai's definitions and use the concept of *revealed favorability* in the following sense: If $x^j \in B^i$ and $B^i \neq B^j$, we say that budget B^i is *revealed more favorable* than budget B^j. In this case, the pair $\{B^i, B^j\}$ is an element of the revealed favorability relation $F \subseteq \mathscr{B} \times \mathscr{B}$. For brevity, we write $B^i F B^j$. That is, given a set of observations on a consumer, we define the relation F as $B^i F B^j$ if $x^j \in B^i$ and $B^i \neq B^j$. Let F^* be the *transitive closure* of the relation F.

Let $\mathscr{F}$ denote the set of all orderings over $\mathscr{B}$. We will denote the elements of $\mathscr{F}$ as $\succsim^F$. The asymmetric part of $\succsim^F$ will be denoted $\succ^F$, i.e. $B^i \succ^F B^j$ if and only if $B^i \succsim^F B^j$ and [not $B^j \succsim^F B^i$].

The *weak axiom of revealed favorability* (WARF) asserts that F is asymmetric: For all $B, B' \in \mathscr{B}$, $B F B'$ implies [not $B' F B$]. The *strong axiom of revealed favorability* (SARF) asserts that the transitive closure of F, F^*, is asymmetric: $B F^* B'$ implies [not $B' F^* B$].

Note again that the revealed favorability relation F is empirically defined.

[3] Sakai (1977) calls the relations on the price-income space *revealed favorability relations* and defines weak and strong axioms of revealed favorability by analogy with WARP and SARP. Little (1979) calls his relations *indirect preference relations* and employs the Congruence Axiom due to Richter (1966). See also Varian (1982), who explores the possibilities of ordinal comparisons between budgets in empirical analysis.

2.3 Duality Relation between Revealed Preference and Revealed Favorability

As the relation between the direct revealed preference relation and the indirect revealed favorability relation is a crucial part of the proof of Theorem 2, it is necessary to briefly state the results of the third section of Sakai (1977).

While WARP excludes multi-valued demand, WARF does not. But WARF does imply that for every $x \in X$, the inverse image of x by h is single-valued, i.e. there can be only one budget at which x is demanded, which is not implied by WARP. To see this, note that $h(B) = \{x, x'\}$ with $x \neq x'$ requires $\{x, x'\} \subset B$ and implies $x R x'$ and $x' R x$ which violates WARP; and that $h^{-1}(x) = \{B, B'\}$ with $B \neq B'$ requires $x \in B \cap B'$ and implies $B F B'$ and $B' F B$ which violates WARF. Thus, WARF does not imply and is not implied by WARP.

Similarly, SARF does not imply and is not implied by SARP. But SARF and single-valued demand combined imply SARP, and SARP and a single-valued inverse image of h combined imply SARF. Finally, if h is a bijection (one-to-one correspondence), then SARF holds if and only if SARP holds.

2.4 Stochastic Revealed Preference and its Weak Axiom

Next we recall the relevant part of the concepts used by BDP (1999, 2004).

A *stochastic demand function* (SDF) is a rule g, which, for every normalized price vector $p \in P$ specifies exactly one probability measure q over the class of all subsets of the budget set $B = B(p)$.

Let $q = g(p)$, where g is an SDF, and let A be a subset of a budget set $B(p)$. Then $q(A)$ is the probability that the bundle chosen by the consumer from the budget set $B(p)$ will be in the set A.

A stochastic demand function g is *degenerate* if, for every normalized price vector $p \in P$, there exists $x \in B(p)$ such that, for every subset A of $B(p)$, $x \in A$ implies $q(A) = 1$ and $x \notin A$ implies $q(A) = 0$, where $q = g(p)$.

A stochastic demand function g satisfies the *weak axiom of stochastic revealed preference* (WASRP) if, for all pairs of normalized price vectors p^i and p^j, and for all $A \subseteq B^i \cap B^j$

$$q^i\left(B^i - B^j\right) \geq q^j(A) - q^i(A), \tag{1}$$

where $q^i = g(p^i)$, $q^j = g(p^j)$, $B^i = B(p^i)$ and $B^j = B(p^j)$.

A stochastic demand function which satisfies $q(\bar{B}) = 1$ is called *tight* (BDP 2004). The analysis here is confined to tight demand.

A stochastic demand function g satisfies *rationalizability in terms of stochastic orderings* (RSO) if there exists an additive probability measure r defined on $\mathscr{R}$ such that, for every normalized price vector p and every subset A of $B = B(p)$

$$\begin{aligned} q(A) = r\big[\big\{\succsim^R \in \mathscr{R} : &\text{ there is a unique } \succsim^R -\text{greatest element in } B \\ &\text{and that element is in } A\big\}\big] \end{aligned} \tag{2}$$

or $q(A) = r\big[\big\{\succsim^R \in \mathscr{R} : \arg\max_{x \in B} \succsim^R \in A\big\}\big]$ for short, where $q = g(p)$.

BDP (1999) show that RSO implies but is not implied by WASRP.

Note that RSO, as defined by BDP, requires that there exists a probability measure over orderings which have a *unique* greatest element in every budget. To illustrate the importance, consider a consumer with a degenerate SDF which satisfies RSO, and identify this SDF with a deterministic demand function. This deterministic demand function has to be single-valued;

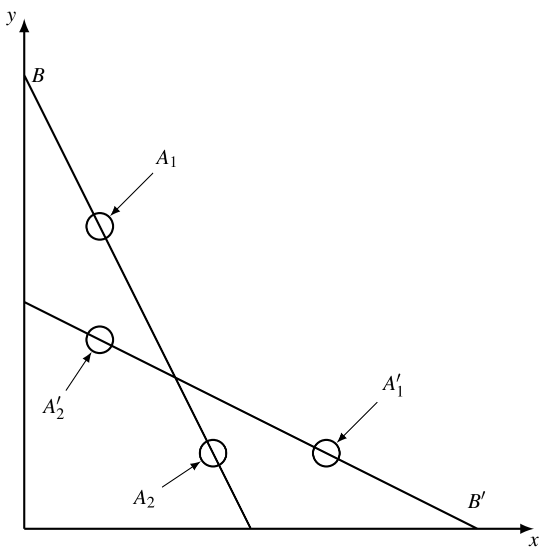

Fig. 1 An example of observed probability assignments on subsets of the budgets B and B': The consumer's stochastic demand function specifies a positive probability only for elements of the indicated subsets of the two budgets. See the text for the numbers and the interpretation.

otherwise, we would have $q(x) = 1$ and $q(y) = 1$ for some $\{x, y\} \subset B$, $x \neq y$, and thus $q(B) \geq 2$, so q cannot be an (additive) probability measure. To allow for a *stochastic demand correspondence*, the additivity property needs to be abandoned, as does Alcantud (2006), who defines his stochastic demand correspondence as a *subadditive* monotone function.

2.5 Indirect Preferences and Stochastic Choice

When we observe probability measures over all subsets of given budgets it is difficult to interpret these measures in terms of preference relations between elements of X, but it appears that a stronger condition than WASRP in analogy to SARP does indeed require explicit usage of transitive closures of preference relations. However, it is more obvious how to interpret the observations in terms of indirect preference relations or revealed favorability relations between elements of $\mathscr{B}$: We can interpret $q^j(B^j \cap B^i)$ as the minimal share of the consumer's indirect preference relations which rank budget B^i over budget B^j.

Consider Figure 1. Suppose on budget B the consumer assigns the probabilities $q(A_1) = 6/8$ and $q(A_2) = 2/8$ to the sets A_1 and A_2 respectively. On budget B' he assigns the probabilities $q'(A'_1) = 4/8$ and $q'(A'_2) = 4/8$. Clearly he reveals that at least $2/8$ of his preference orderings rank B' over B, and at least $4/8$ of his preference orderings rank B over B'.

Now suppose the indicated subsets of the budgets are singletons. The observed probability assignments are consistent with a consumer who has three different preferences $\succsim_a^R$, $\succsim_b^R$, $\succsim_c^R$, such that A_1 is the $\succsim_a^R$-greatest element of B and A'_2 is the $\succsim_a^R$-greatest element of B' and the preference $\succsim_a^R$ is realized with probability $4/8$; A_1 is the $\succsim_b^R$-greatest element of B and A'_1 is the $\succsim_b^R$-greatest element of B' and the preference $\succsim_b^R$ is realized with probability $2/8$; A_2 is the $\succsim_c^R$-greatest element of B and A'_1 is the $\succsim_c^R$-greatest element of B' and the preference $\succsim_c^R$ is realized with probability $2/8$.

When we try to find a probability assignment over indirect preferences, the only (necessary and sufficient) conditions imposed by the observed probability assignments on the probabilities are that the consumer has an indirect preference which ranks budget B over B' and which is realized with a probability of at least $4/8$, and an indirect preference which ranks budget B' over B and which is realized with a probability of at least $2/8$. For example the consumer could have two different indirect preferences $\succ_a^F$ and $\succ_b^F$, such that B is the $\succ_a^F$-greatest element of $\{B, B'\}$ and the preference $\succ_a^F$ is realized with probability $5/8$; B' is the $\succ_b^F$-greatest element of $\{B, B'\}$ and the preference $\succ_b^F$ is realized with probability $3/8$.

3 Rationalizability

3.1 Rationalizability in Terms of Stochastic Orderings on the Normalized Price Space

The idea of the notion of rationalizability considered in this section is that there exists a probability measure on the set of indirect preferences which generates the observed choices. More formally, let $N = \{1, 2, \ldots, n\}$ be the set of indices of the observed budgets, and let $M \subset N$ with some index $k \in N$ and $k \notin M$. We say that a stochastic demand function g satisfies *rationalizability in terms of stochastic orderings on the normalized price space* (RSOP) if there exists a probability measure f defined on $\mathscr{F}$ such that f generates the observed stochastic demand in following sense: For all $k \in N$ and all $M \subseteq N - \{k\}$,

$$f\left[\left\{\succ^F \in \mathscr{F} : (\forall i \in M)\,[B^i \succ^F B^k]\right\}\right] \geq q^k\Big(B^k \cap \bigcap_{i \in M} B^i\Big), \tag{3a}$$

$$f\left[\left\{\succ^F \in \mathscr{F} : (\forall i \in M)\,[B^k \succ^F B^i]\right\}\right] \leq q^k\Big(B^k - \bigcup_{i \in M} B^i\Big). \tag{3b}$$

That is, f satisfies Eq. (3a) and (3b): The sum over all indirect preferences which rank all budgets in $\{B^i\}_{i \in M}$ higher than B^k is greater than or equal to the choice probability assigned to the part of B^k that intersects with all $\{B^i\}_{i \in M}$. The sum over all indirect preferences which rank all budgets in $\{B^i\}_{i \in M}$ lower than B^k is less than or equal to the choice probability assigned to the part of B^k that does not intersect with any B^i in $\{B^i\}_{i \in M}$.

Note that RSOP is defined over asymmetric orderings $\succ^F \in \mathscr{F}$ and not over $\succsim^F \in \mathscr{F}$ (when an ordering $\succsim^F$ is linear, it is identical to its asymmetric part $\succ^F$). Allowing for indifference between distinct budget would at the very least complicate the issue substantially. Furthermore, the definition is justified by the analogy between SARP and SARF. While SARP excludes indifference between two distinct bundles of a single budget, SARF does not. Instead, SARF excludes indifference between two distinct budgets, while SARP does not. Similarly, RSO requires the existence of a probability measure over orderings which have a unique greatest element in a budget.

We need inequalities $\geq$ and $\leq$ rather than equality $=$ in Eq. (3a) and (3b) because, as the example in Section 2.5 illustrates, the observed stochastic demand only imposes upper and lower bounds on the probability measure over indirect preferences. For example, $q^k(B^k \cap B^i)$ requires that the probability of all $\succ^F \in \mathscr{F}$ such that $B^i \succ^F B^k$ equals *at least* $q^k(B^k \cap B^i)$; in Section 2.5 we had $q(A_2) = q(B \cap B') = 2/8$, but $f[\{\succ^F \in \mathscr{F} : B' \succ^F B]\}] = 3/8$. Also note that if the inequalities were replaced by equalities, the measure f may not be a probability measure because we can have, say, $q(B - B') = q'(B' - B) = 1$, which is clearly rationalizable but would require $f[\{\succ^F \in \mathscr{F} : B \succ^F B'\}] = f[\{\succ^F \in \mathscr{F} : B' \succ^F B\}] = 1$.

To illustrate RSOP, consider the example given in Section 2.5. It can be easily checked that RSOP is satisfied and that the assignment of probabilities in the example can also be assigned under RSOP. Suppose that instead of the numbers in Section 2.5 we have $q(A_1) = q'(A'_1) = 1/3$ and $q(A_2) = q'(A'_2) = 2/3$. Then RSOP requires that $f[\{\succ^F \in \mathscr{F} : B \succ^F B'\}] \geq q'(B' \cap B) = 2/3$ and $f[\{\succ^F \in \mathscr{F} : B \succ^F B'\}] \leq q(B - B') = 1/3$, which is impossible. Note that the consumer reveals that he prefers budget B over budget B' with probability $2/3$ and also that he prefers budget B' over budget B with probability $2/3$, a contradiction in the sense of RSOP.

The axiom RSOP is based on the restrictions imposed on a deterministic demand function consistent with an ordering of the n budgets in the sense of SARF, i.e. an asymmetric ordering. We say that an ordering $\succ^F$ is *possible* if the demand function is consistent with the ordering, i.e. if given the observed demand function the hypothesis that the consumer acts according to $\succ^F$ cannot be rejected. Similarly, we say that an ordering $\succ^F$ is *necessary* if the demand function is only consistent with that ordering.

Recall the definition of h as a non-empty demand function on $\mathscr{B}$ (Section 2.1). The following proposition will be helpful for the proof of Theorem 3.2 below.

Proposition 1

1. *An ordering $B^1 \succ^F B^2 \succ^F \cdots \succ^F B^n$ is possible if and only if*

$$h(B^1) \notin \bigcup_{i \in \{2,3,\ldots,n\}} B^i,\ h(B^2) \notin \bigcup_{i \in \{3,4,\ldots,n\}} B^i, \ldots,\ B^{n-1} \notin B^n. \tag{4}$$

2. *An ordering $B^1 \succ^F B^2 \succ^F \cdots \succ^F B^n$ is necessary if and only if*

$$h(B^n) \in B^{n-1},\ h(B^{n-1}) \in B^{n-2}, \ldots,\ h(B^2) \in B^1. \tag{5}$$

Proof

1. Because the ordering $\succ^F$ is a complete ranking of all budgets, the condition Eq. (4) means that no other budget is neither directly nor indirectly preferred to B^1. Hence the observation cannot be used to reject $B^1 \succ^F B^i$ for all $i \in \{2,3,\ldots,n\}$, and similarly for $h(B^2) \notin \bigcup_{i \in \{3,4,\ldots,n\}}$ etc. If $h(B^1) \in B^i$ for some $i \in \{2,3,\ldots,n\}$, then we must have $B^i\, F, B^1$, and similarly for $h(B^2)$ etc.
2. Note that $h(B^i) \in B^j$ implies $B^j\, F\, B^i$. If also $h(B^j) \in B^k$ then $B^k\, F^*\, B^i$ etc. The “only if” part can be shown analogously to the proof of the “if” part in step 1.

Note that RSOP can be defined in terms of the complements of the sets used in the original definition above. In fact, it is equivalent to a definition in terms of the complements:

Proposition 2 RSOP *is equivalent to*

$$f\left[\left\{\succ^F \in \mathscr{F} : (\exists i \in M)\, [B^k \succ^F B^i]\right\}\right] \leq q^k\Big(B^k - \bigcap_{i \in M} B^i\Big) \tag{6a}$$

$$f\left[\left\{\succ^F \in \mathscr{F} : (\exists i \in M)\, [B^i \succ^F B^k]\right\}\right] \geq q^k\Big(B^k \cap \bigcup_{i \in M} B^i\Big). \tag{6b}$$

More specifically, (6a) is equivalent to (3a) and (6b) is equivalent to (3b).

Proof Note that $\left(B^k - \bigcap_{i\in M} B^i\right) \cap \left(B^k \cap \bigcap_{i\in M} B^i\right) = \emptyset$ and $\left(B^k - \bigcap_{i\in M} B^i\right) \cup \left(B^k \cap \bigcap_{i\in M} B^i\right) = B^k$, so we have

$$q^k\Big(B^k - \bigcap_{i\in M} B^i\Big) = 1 - q^k\Big(B^k \cap \bigcap_{i\in M} B^i\Big);$$

and similarly

$$f\left[\left\{\succ^F \in \mathscr{F} : (\exists i \in M)\,[B^k \succsim^F B^i]\right\}\right] = 1 - f\left[\left\{\succ^F \in \mathscr{F} : (\forall i \in M)\,[B^i \succsim^F B^k]\right\}\right],$$

which proves the equivalence of (6a) and (3a). The proof works analogously for the equivalence of (6b) and (3b).

Because the number of different indirect preferences is finite if the number of observations is finite, it is straightforward to test for RSOP. Let $S(N)$ be the set of all ordered n-tuples of indices in N, i.e. the set of the $n!$ permutations of N. The elements of $S(N)$ will be indicated by σ, and more explicitly as $\sigma_i = \langle a, b, \ldots, e\rangle$ and $\sigma_i(1) = a$, $\sigma_i(2) = b$ etc., and $\sigma_i^{-1}(a) = 1$, $\sigma_i^{-1}(b) = 2$ etc. Let $\pi_i = \pi(\sigma_i)$ be the probability assigned to the ordering σ_i.

We now define the following linear feasibility problem:

$$\text{find } \Pi = (\pi_1, \pi_2, \ldots, \pi_{n!}) \tag{FP.1}$$

$$\text{satisfying } \pi_i \geq 0 \ \text{ for all } i = 1, 2, \ldots, n! \tag{FP.2}$$

$$\sum_{i=1}^{n!} \pi_i = 1 \tag{FP.3}$$

$$\sum_{\left\{i\in\{1,\ldots,n!\}:\sigma_i^{-1}(j)<\sigma_i^{-1}(k)\forall j\in M\right\}} \pi_i \geq q^k\Big(B^k \cap \bigcap_{j\in M} B^j\Big) \tag{FP.4}$$

$$\sum_{\left\{i\in\{1,\ldots,n!\}:\sigma_i^{-1}(j)>\sigma_i^{-1}(k)\forall j\in M\right\}} \pi_i \leq q^k\Big(B^k - \bigcup_{j\in M} B^j\Big) \tag{FP.5}$$

$$\text{for all non-empty } M \subset N \text{ and all } k \in N, k \notin M$$

Note that $\sum_{\left\{i\in\{1,\ldots,n!\}:\sigma_i^{-1}(j)<\sigma_i^{-1}(k)\forall j\in M\right\}} \pi_i$ denotes the sum over all probability assignments over preferences which rank *all* $j \in M$ higher than k, excluding preferences which rank one or more $j \in M$ lower than k.

Theorem 1 *The following conditions are equivalent:*

(1) there exists a probability measure f over the set of all orderings on $\mathscr{B}$ that rationalizes the stochastic choices $\left\{q(B^i)\right\}_{i=1}^n$, i.e. RSOP *is satisfied;*

*(2) the linear feasibility problem (*FP*) has a solution.*

Proof Because RSOP is defined over asymmetric orderings which excludes indifference, the equivalence is obvious.

Note that the qs used in (FP) are specified by the stochastic demand function g; thus, (FP) is indeed a testable condition on the stochastic demand function.

3.2 Rationalizability in Terms of Stochastic Orderings on the Commodity Space

Sakai (1977,Theorem 6) shows that if the (deterministic) demand at every normalized price vector is a singleton and the demand function satisfies SARF, then a (direct) utility function can be deduced from the favorability relation. Similarly, we arrive at the following interesting theorem.

Theorem 2 *If* RSOP *is satisfied, then the stochastic demand function g satisfies rationalizability in terms of stochastic orderings (*RSO*).*

Proof By convenience we define functions $g_{\succsim^R} : \mathscr{B} \to X$ such that $g_{\succsim^R}(B) = \arg\max_{x \in B} \succsim^R$ for all $\succsim^R \in \mathscr{R}$ which have a unique greatest element in every $B \in \mathscr{B}$. We say that an ordering $\succsim^R \in \mathscr{R}$ over X is *consistent* with an ordering $\succ^F \in \mathscr{F}$ over $\mathscr{B}$ if $g_{\succsim^R}$ is defined and $g_{\succsim^R}(B) \succsim^R g_{\succsim^R}(B')$ whenever $B \succ^F B'$. Let $\mathscr{Q}$ be the subset of $\mathscr{R}$ such that all $\succsim^R \in \mathscr{Q}$ are consistent with a $\succ^F \in \mathscr{F}$. That $\mathscr{Q}$ is not empty follows directly from Sakai (1977, Theorem 6) and the possibility to extend an asymmetric revealed preference relation to a complete ordering (cf. Newman 1960). Then $g_{\succsim^R}$ is defined if $\succsim^R \in \mathscr{Q}$.

In analogy to the definitions of possibility and necessity of $\succ^F$, we say that an ordering $\succsim^R$ is *possible* if given the observed demand function the hypothesis that the consumer acts according to $\succsim^R$ cannot be rejected, and that an ordering $\succsim^R$ is *necessary* if given the observed demand function the hypothesis that the consumer acts according to $\succsim^R$ cannot be rejected.

Lemma 1 *For a deterministic demand function, if $\succ^F$ is possible, then so is every $\succsim^R$ which is consistent with $\succ^F$. If $\succsim^R$ is necessary and consistent with $\succ^F$, then $\succ^F$ is also necessary.*

Proof Suppose $\succ^F$ is such that $B^1 \succ^F B^2 \succ^F \cdots \succ^F B^n$. Then if $\succ^F$ is consistent with $\succsim^R$, we have $g_{\succsim^R}(B^1) \succsim^R g_{\succsim^R}(B^2) \succsim^R \cdots \succsim^R g_{\succsim^R}(B^n)$. It is easy to show that this $\succsim^R$ is possible if the same condition as is Proposition 1.(1) is satisfied, and that $\succsim^R$ is necessary only if the same condition as in Proposition 1.(2) is satisfied.[4]

Let η be a probability measure over the set of single-valued deterministic demand functions H. Let s be a probability measure over $\mathscr{R}$. Then by Lemma 1,

$$s\big[\big\{\succsim^R \in \mathscr{Q} : g_{\succsim^R}(B^1) \succsim^R g_{\succsim^R}(B^2) \succsim^R \cdots \succsim^R g_{\succsim^R}(B^n)\big\}\big] = \lambda \in [0,1] \tag{7}$$

is possible if

$$\begin{aligned}
&\eta\left[\left\{h \in H : h(B^1) \in B^1 - \bigcup_{j \in \{2,3,\ldots,n\}} B^j\right\}\right] \geq \lambda, \\
&\eta\left[\left\{h \in H : h(B^2) \in B^2 - \bigcup_{j \in \{3,4,\ldots,n\}} B^j\right\}\right] \geq \lambda, \\
&\ldots, \\
&\eta\big[\big\{h \in H : h(B^{n-1}) \in B^{n-1} - B^n\big\}\big] \geq \lambda.
\end{aligned}$$

This condition is satisfied if RSOP is satisfied, and analogously for necessity Eq. (7).

[4] Note that $\succsim^R$ is possible *if*, but not *only if*, $\succ^F$ is possible, and that $\succsim^R$ is necessary *only if*, but not necessarily *if*, $\succ^F$ is necessary.

Identify the probability measure η over H with the probability measure q which is specified by an SDF, i.e. $\eta[\{h \in H : h(B^i) \in A\}] = q^i(A)$. Then, with Lemma 1, we can assign probabilities over $\mathscr{Q}$ induced by f, i.e.

$$r\big[\big\{\succsim^R \in \mathscr{Q} : (\forall i \in M)\,[g_{\succsim^R}(B) \succsim^R g_{\succsim^R}(B^i)]\big\}\big] := f\left[\big\{\succ^F \in \mathscr{F} : (\forall i \in M)\,[B \succ^F B^i]\big\}\right], \tag{8a}$$

$$r\big[\big\{\succsim^R \in \mathscr{Q} : (\forall i \in M)\,[g_{\succsim^R}(B^i) \succsim^R g_{\succsim^R}(B)]\big\}\big] := f\left[\big\{\succ^F \in \mathscr{F} : (\forall i \in M)\,[B^i \succ^F B]\big\}\right], \tag{8b}$$

where r is an additive probability measure over $\mathscr{Q} \subset \mathscr{R}$ as required by RSO.

We then have that

$$\Big\{\succsim^R \in \mathscr{Q} : g_{\succsim^R}(B^k) \in B^k \cap \bigcap_{i\in M} B^i\Big\} \subseteq \big\{\succsim^R \in \mathscr{Q} : (\forall i \in M)\,[g_{\succsim^R}(B^i) \succsim^R g_{\succsim^R}(B^k)]\big\}, \tag{9a}$$

$$\Big\{\succsim^R \in \mathscr{Q} : g_{\succsim^R}(B^k) \in B^k - \bigcup_{i\in M} B^i\Big\} \supseteq \big\{\succsim^R \in \mathscr{Q} : (\forall i \in M)\,[g_{\succsim^R}(B^k) \succsim^R g_{\succsim^R}(B^i)]\big\}, \tag{9b}$$

and therefore

$$r\Big[\Big\{\succsim^R \in \mathscr{Q} : g_{\succsim^R}(B^k) \in B^k \cap \bigcap_{i\in M} B^i\Big\}\Big] \le r\big[\big\{\succsim^R \in \mathscr{Q} : (\forall i \in M)\,[g_{\succsim^R}(B^i) \succsim^R g_{\succsim^R}(B^k)]\big\}\big],$$

$$r\Big[\Big\{\succsim^R \in \mathscr{Q} : g_{\succsim^R}(B^k) \in B^k - \bigcup_{i\in M} B^i\Big\}\Big] \ge r\big[\big\{\succsim^R \in \mathscr{Q} : (\forall i \in M)\,[g_{\succsim^R}(B^k) \succsim^R g_{\succsim^R}(B^i)]\big\}\big].$$

Eq. (9a) and (9b) follow from Lemma 1 and the restriction to $\succsim^R \in \mathscr{Q}$.

Suppose that $A \subseteq B^k \cap \bigcap_{i\in M} B^i$ for some $M \subset N$ and $k \in N - M$. We have

$$q^k\Big(\Big(B^k \cap \bigcap_{i\in M} B^i\Big) - A\Big) + q^k(A) = q^k\Big(B^k \cap \bigcap_{i\in M} B^i\Big).$$

If

$$q^k\Big(B^k \cap \bigcap_{i\in M} B^i\Big) = r\Big[\Big\{\succsim^R \in \mathscr{Q} : g_R(B^k) \in B^k \cap \bigcap_{i\in M} B^i\Big\}\Big],$$

then Eq. (9a) and the fact that

$$\Big\{\succsim^R \in \mathscr{Q} : g_{\succsim^R}(B^k) \in B^k \cap \bigcap_{i\in M} B^i - A\Big\} \cup \big\{\succsim^R \in \mathscr{Q} : g_{\succsim^R}(B^k) \in A\big\}$$
$$= \Big\{\succsim^R \in \mathscr{R} : g_{\succsim^R}(B^k) \in B^k \cap \bigcap_{i\in M} B^i\Big\}$$

tell us that $r[\{\succsim^R \in \mathscr{Q} : g_{\succsim^R}(B^k) \in A\}] = q^k(A)$ meets the requirement of RSOP and is consistent with RSO.

Note that $g_{\succsim^R}(B^k) \in B^k \cap \bigcap_{i\in M} B^i$ is possible if $(\forall i \in M)\,[g_{\succsim^R}(B^i) \succsim^R g_{\succsim^R}(B^k)]$. By Lemma 1 $(\forall i \in M)\,[g_{\succsim^R}(B^i) \succsim^R g_{\succsim^R}(B^k)]$ is possible if $(\forall i \in M)\,[B^i \succ^F B^k]$. Thus, if $(\forall i \in M)\,[B^i \succ^F B^k]$ then $g_{\succsim^R}(B^k) \in B^k \cap \bigcap_{i\in M} B^i$ is possible. By RSOP (Eq. 3a) we have

$$q^k\Big(B^k \cap \bigcap_{i\in M} B^i\Big) \le f\big[\big\{\succ^F \in \mathscr{F} : (\forall i \in M)\,[B^i \succ^F B^k]\big\}\big],$$

and thus with

$$r\left[\left\{\succsim^R\in\mathscr{Q}:g_{\succsim^R}(B^k)\in B^k\cap\bigcap_{i\in M}B^i\right\}\right]=q^k\left(B^k\cap\bigcap_{i\in M}B^i\right)$$

we have

$$r\left[\left\{\succsim^R\in\mathscr{Q}:g_{\succsim^R}(B^k)\in B^k\cap\bigcap_{i\in M}B^i\right\}\right]\le f\left[\left\{\succ^F\in\mathscr{F}:(\forall i\in M)\left[B^i\succ^F B^k\right]\right\}\right].$$

Similarly is can be shown that with RSOP, for every $A\subseteq B^k-\bigcup_{i\in M}B^i$

$$r[\{\succsim^R\in\mathscr{Q}:g_{\succsim^R}(B^k)\in A\}]=q^k(A)$$

and

$$r\left[\left\{\succsim^R\in\mathscr{Q}:g_{\succsim^R}(B^k)\in B^k-\bigcup_{i\in M}B^i\right\}\right]=q^k\left(B^k-\bigcup_{i\in M}B^i\right)$$

is possible.

Note that RSOP excludes indifference between distinct budgets, while RSO does not. Thus, RSO does not imply RSOP: Consider a degenerate stochastic demand function on two budgets $B^1\neq B^2$ and $x\in B^1\cap B^2$ with $q^1(x)=q^2(x)=1$, so RSO is satisfied while RSOP is not.

Remark 1 Recall that if a deterministic demand function is a bijection, SARP and SARF are equivalent (Section 2.3, Sakai 1977, p. 117). Similarly, can be shown that if the probability measure η can be restricted to a subset $H'\subset H$ such that (i) each $h\in H'$ is a bijection, and (ii) $\eta(H')=1$, and this measure η is identified with the probability measure q which is specified by an SDF, then RSOP and RSO are equivalent. Informally, if h is a bijection and identified with a degenerate stochastic demand function, the example given in the last paragraph of the preceding proof loses its relevance, as $h(B^1)=h(B^2)=x$ is not possible any more. If restricted to bijective demand functions, the equivalent of Proposition 1 can be shown to hold for $\succsim^R$. Lemma 1 then becomes strong enough to show the equivalence between RSOP and RSO. As Maharam and Stone (1982) show, the set of bijective functions is dense in the space of all functions. Thus, every function can arbitrarily close be approximated by a bijective function. A detailed analysis of the implications for the framework considered here is beyond the scope of the article, but future work will focus on how an injective or surjective demand function can be approximated by bijections in order to show how close the direct and indirect approaches in general are.

3.3 Problems and Open Questions

Consider the following construction: A budget B^i is *revealed more favorable by degree* $\varphi(i,j)$ than B^j if

$$\begin{aligned}\varphi(i,j)=\max\Bigg\{q^j\left(B^j\cap B^i\right),\ q^j&\left(B^j\cap B^{M(1)}\right)\\&+\sum_{k=1}^{m-1}q^{M(k)}\left(B^{M(k)}\cap B^{M(k+1)}\right)\\&+q^{M(m)}\left(B^{M(m)}\cap B^i\right)-m\Bigg\},\end{aligned}\tag{10}$$

where the maximum is over all sets of indices $M \subseteq N - \{i, j\}$. Then obviously

$$\varphi(i,j) + \varphi(j,i) \leq 1 \tag{11}$$

is a necessary condition for RSOP. "Revealed more favorable by degree" means that the probability that a consumer has preferences according to which a budget i is preferred over a budget j is at least $\varphi(i,j)$. Obviously, we need $q^j\left(B^j \cap B^i\right) \leq \varphi(i,j)$; the second part of the definition attempts to extend this to indirect revelations. It may seem to be a reasonable conjecture that the condition is also sufficient, but unfortunately it is not, as will be shown below. The problem is that the stochastic demand of a consumer can imply a higher probability that i is preferred over j than is captured by the definition of $\varphi(i,j)$.

But first note the following:

Proposition 3 *Identify a deterministic demand function with a degenerate stochastic demand function. For that demand function, condition (11) is equivalent to the strong axiom of revealed favorability.*

Proof In the deterministic case, $B^i F^* B^j$ is equivalent to $\varphi(i,j) = 1$. To see this, note that (i) $\varphi(i,j) \in \{0,1\}$, (ii) $B^i F B^j$ is equivalent to $q^j\left(B^j \cap B^i\right) = 1$, and (iii) $B^i F^* B^j$ is equivalent to $q^j\left(B^j \cap B^{M(1)}\right) = 1$, $q^{M(1)}\left(B^{M(1)} \cap B^{M(2)}\right) = 1, \ldots, q^{M(m)}\left(B^{M(m)} \cap B^{M(i)}\right) = 1$ for some $M \subset N$. So condition (11) is equivalent to asymmetry of F^*.

A "system of binary probabilities" $[\alpha_{ij} : i,j \in \{1,2,\ldots,n\}, i \neq j, \alpha_{ij} + \alpha_{ji} = 1]$ is said to be "induced by rankings" (rationalizable) if there is a probability distribution on the set of $n!$ orderings of $\{1,2,\ldots,n\}$ such that, for all distinct i and j, α_{ij} is the sum of all probabilities attached to orderings which rank i over j (cf. Fishburn 1990). The so-called *triangular condition*

$$\alpha_{ij} + \alpha_{jk} + \alpha_{ki} \leq 2 \tag{12}$$

and its generalization

$$\alpha_{M(1)M(2)} + \alpha_{M(2)M(3)} + \ldots + \alpha_{M(m)M(1)} \leq m - 1 \tag{13}$$

for all sets of indices $M \subseteq N$ of length m is a necessary condition for rationalizability.[5] It was also conjectured to be a sufficient condition for rationalizability by Marschak (1959). In an unpublished article, McFadden and Richter (1970) (see also McFadden 2005) provided a counterexample for $n = 6$. Later on, Fishburn (1990) observed that the set of conditions on the choice probabilities that are sufficient for rationalizability regardless of n must be infinite.

This poses some problems for the framework considered here. Consider the counterexample of McFadden and Richter (1970) applied to the framework of stochastic revealed preference: For $n = 6$, let

$$\alpha_{12} = \alpha_{14} = \alpha_{34} = \alpha_{36} = \alpha_{56} = \alpha_{52} = 1$$
$$\alpha_{21} = \alpha_{41} = \alpha_{43} = \alpha_{63} = \alpha_{65} = \alpha_{25} = 0$$
$$\alpha_{ij} = 1/2 \text{ for all other } i, j$$

[5] For the generalized form, see for example Cohen and Falmagne (1990). In the case of binary probabilities, the generalized form can be deduced from the triangular condition.

where $q^j\left(B^j \cap B^i\right) = \alpha_{ij}$. Then the triangular condition and its generalization are satisfied, and so is condition (11); but (FP) has no solution. Indeed, with $q^j\left(B^j \cap B^i\right) = \alpha_{ij}$, conditions (13) implies (11) because

$$\begin{aligned}\varphi(i,j) + \varphi(j,i) = \alpha_{jM^i(1)} &+ \alpha_{M^i(1)M^i(2)} + \ldots + \alpha_{M^i(m)i} \\ &+ \alpha_{iM^j(1)} + \alpha_{M^j(1)M^j(2)} + \ldots + \alpha_{M^j(m)j} \\ &- m^i - m^j,\end{aligned} \tag{14}$$

where M^i and M^j, $|M^i| = m^i$ and $|M^j| = m^j$, are the sets of indices used to construct $\varphi(i,j)$ and $\varphi(j,i)$, and with (14) and condition (13) we obtain

$$\begin{aligned}\varphi(i,j) + \varphi(j,i) + m^i + m^j &\leq (m^i + 1) + (m^j + 1) - 1 \\ \Leftrightarrow \varphi(i,j) + \varphi(j,i) &\leq 1.\end{aligned}$$

While it might also be possible that exploitation of the particularities of the framework of BDP, e.g. linearities of budgets, helps to find finite sets of necessary and sufficient conditions for stochastic revealed preference without applicability to the binary probability problem, the results of this section suggest that conditions for RSOP based on definitions for a partial revealed favorability relation between budgets suffer from similar problems as the conditions for rationalizability of binary probabilities. Therefore a “strong axiom of stochastic revealed favorability” could possibly also solve the problem of finding a finite set of necessary and sufficient conditions for systems of binary probabilities for each particular n.

4 Conclusion

The weak axiom of stochastic revealed preference, as introduced by Bandyopadhyay, Dasgupta, and Pattanaik (1999), is a necessary but not sufficient condition for stochastic demand behavior to be rationalizable in terms of stochastic orderings on the commodity space. It was the purpose of this article to explore rationalizability issues and to show how one can, in principle, test whether or not a finite set of observations of stochastic choice is rationalizable by stochastic orderings.

To this end the problem of finding a probability measure over orderings on the commodity space was transformed into a problem of finding a probability measure over orderings on the normalized price space. The advantage of this indirect approach is that it avoids the problems resulting from the infinity of the set of alternatives a consumer chooses from when facing a budget set defined in the usual way. Furthermore, rationalizability in terms of stochastic orderings on the normalized price space was shown to be a sufficient condition for rationalizability in terms of stochastic orderings on the commodity space.

In Section 3.3 similarities with binary probability systems were pointed out. In particular it was shown that conditions based on partial revealed favorability relations are likely to suffer from similar problems as the conditions for rationalizability of binary probabilities.

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